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TOTAL PATHOS TOTAL VERTEX SEMIENTIRE BLOCK GRAPH

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ABSTRACT

In this paper, we introduce the concept of total pathos total vertex semientire block graph of a tree. We obtain some properties of this graph. We study the characterization of graphs whose of total pathos total vertex semientire block graph of a tree is Hamiltonian, nonplanar and noneulerian.

KEYWORDS: Block Graph, Inner Vertex Number, Line Graph, Path of Pathos, Vertex Semientire Graph

Mathematics Subject Classification: 05C

1. INTRODUCTION

All graphs considered here are finite, undirected without loops or multiple edges. Any undefined term or notation in this paper may be found in Harary [2].

The concept of pathos of a graph G was introduced by Harary [2], as a collection of minimum number of line disjoint open paths whose union is G. The path number of a graph G is the number of paths in pathos.

For a graph G(p, q) if B = { u_1, u_2, u_3, · · · , u_r; r ≥2} is a block of G , then we say that point u_1 and block B are incident with each other, as are u_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut vertex then they are called adjacent blocks.

By a plane graph G we mean embedded in the plane as opposed to a planar graph. In a plane graph G, let e_1={u,v }be an edge. We say e_1 is adjacent to the vertices u and v, which are also adjacent to each other. Also an edge e_1 is adjacent to the edge e_2 =uw. A region of G is adjacent to the vertices and edges which are on its boundary, and two regions of G are adjacent if their boundaries share a common vertex.

The *edgedegree* of an edge e = {a, b} is the sum of degrees of the end vertices a and b. Degree of a block is the number of vertices lies on a block. *Blockdegree* B_v of a vertex v is the number of blocks in which v lies. Degree of a region is the number of vertices lies on a region. The *regiondegree* R_v of a vertex v is the number of regions in which the vertex v lies.

A new concept of a graph valued functions called the vertex semientire block graph $e_{vb}(G)$ of a plane graph G was introduced by Venkanagouda in [9] and is defined as the graph whose vertex set is the union of the set of vertices, blocks and regions of a graph G in which two vertices are adjacent if and only if they are adjacent vertices, vertices lie on the blocks and vertices lie on the regions.

The total vertex semientire block graph $T_e(G)$ of a plane graph G is the graph whose vertex set is the union of the set of vertices, blocks and regions of a graph G in which two vertices are adjacent if the corresponding vertices or blocks of G are adjacent or one corresponding to the vertex v of G and other to a blocks b of G and v is in b or one corresponds to the vertex v of G and other the region r of G and v lies on the region r. This concept was introduced by Venkanagouda in [6].

The pathos total vertex semientire block graph $P_{Tv}(T)$ of tree T is the graph whose vertex set is the union of the set of vertices, blocks, regions and path of pathos of a tree T in which two vertices are adjacent if the corresponding vertices or blocks of T are adjacent or one corresponding to the vertex v of T and other to a blocks b of T and v is in b or one corresponds to the vertex v of T and other the region r of T and v lies on the region r or one corresponds the vertex v of T and the other the path of pathos p of T and v lies on p. This concept was introduced by Rajanna and Venkanagouda in [10].

Total pathos vertex semientire graph of a tree T denoted by $T_{pv}(T)$ is the graph whose vertex set is the union of the set of vertices, blocks, regions and the path of pathos of a tree T in which two vertices are adjacent if the corresponding vertices of T are adjacent or one corresponding to the vertex v of T and other to a blocks b of T and v is in b or one corresponds to the vertex v of T and other the region r of T and v lies in the region r in T or one corresponds the vertex v of T and the other the path of pathos p of T and v lies on p or both are the path of pathos p_i and p_j and have a common vertex. The total pathos vertex semientire block graph is defined only when the tree contains at least two paths. This concept was introduced by Rajanna and Venkanagouda in [6].

The block graph B(G) of a graph G is the graph whose vertex set is the set of blocks of G in which two vertices are adjacent if the corresponding blocks are adjacent. This graph was studied in [2].

Path graph P(T) of a tree T is the graph whose vertex set is the set of path of pathos of T in which two vertices are adjacent if the corresponding path of pathos have a common vertex.

The following will be useful in the proof of our results.

Theorem 1[4].

If G be a connected plane graph then the vertex semientire graph $e_v(G)$ is planar if and only if G is a tree.

Theorem 2[10].

For any nontrivial tree T, the pathos total vertex semientire block $P_{Tv}(T)$ of a tree T, whose vertices have degree and if is number of blocks to which vertex belongs in T, then $P_{Tv}(T)$ has (2p +k) vertices and edges, where v(pi) be the number of vertices lies on the path p_i **.**

Theorem 3 [10].

For any graph G, the pathos total vertex semientire block graph $P_{Tv}(T)$ is always nonseparable.

Theorem 4 [2].

A nontrivial graph is bipartite if and only if all its cycles are even.

Theorem 5[2].

A connected graph G is eulerian if and only if each vertex in G has even degree.

Theorem 6 [10].

For any tree T, $P_{Tv}(T)$ is always nonplanar.

1. Total Pathos Total Vertex Semientire Block Graph

Now we define the following graph valued function.

The total pathos total vertex semientire block graph $T_{PT}(T)$ of tree T is the graph whose vertex set is the union of the set of vertices, blocks, regions and path of pathos of a tree T in which two vertices are adjacent if the corresponding vertices or blocks or paths of pathos of T are adjacent or one corresponding to the vertex v of T and other to a blocks b of T and v is in b or one corresponds to the vertex v of T and other the region r of T and v lies on the region r or one corresponds the vertex v of T and the other the path of pathos p of T and v lies on p.

In Figure 2.3, a graph G and its total pathos total vertex semientire block graph are shown. Since the path of pathos is not unique, the corresponding total pathos total vertex semientire block graph is also not unique.

Remark 1.

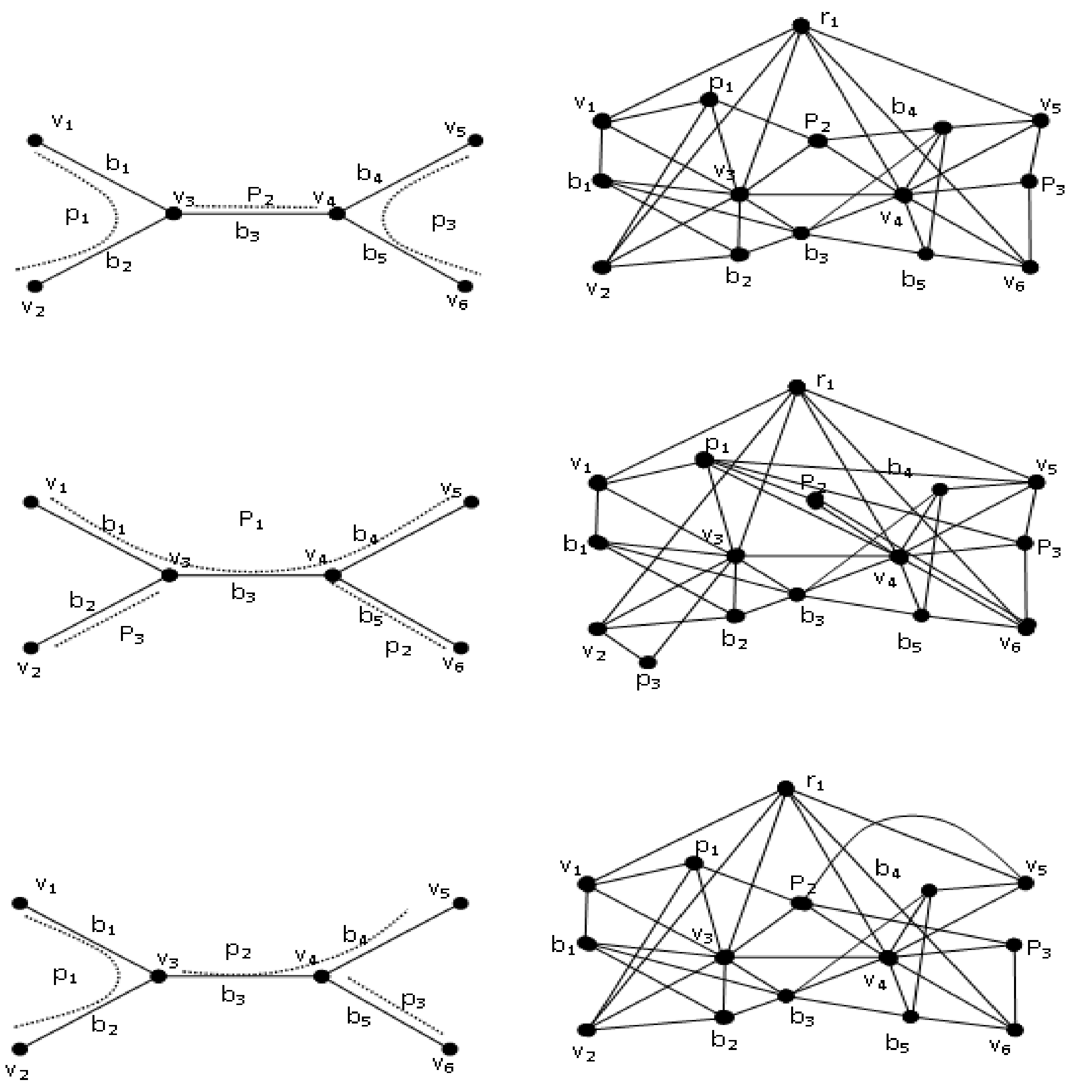
For very tree T, $T_{PT}(T) = P_{TV}(T) \bigcup P(T).$

Remark 2.

For any tree T, $T \subseteq e_{vb}(T) \subseteq \mathrm{T}_{e}(T) \subseteq P_{Tv}(T)$.

Remark 3.

If deg v = n for $v \in T$ then the degree of the corresponding vertex in $T_{PT}(T)$ is $2n + \left\lceil \frac{n}{2} \right\rceil + 1$.



T **Figure 2.3** $T_{PT}(T)$

We now establish a result which determines the number of vertices and edges in total pathos total vertex semientire block graph of a tree T.

Theorem 7.

For any nontrivial tree T, the total pathos total vertex semientire block $T_{PT}(T)$ of a tree T, whose vertices have degree d_i and if b_i is number of blocks to which vertex v_i belongs in T, then $T_{PT}(T)$ has (2p +k) vertices and

$$4q+1+\sum_{1}^{k} v(p_i)+\frac{1}{2}\sum b_i(b_i-1)+\frac{1}{2}\sum \Delta p_i(\Delta p_i-1)$$

edges, where $v(p_i)$ be the number of vertices lies on the path p_i. Δp_i be the number of paths in which v_i lies.

Proof.

By the definition, $T_{PT}(T)$ spanning subgraph of P_{Tv} (T). Thus the number of vertices of $P_{Tv}(T)$ equals to the number of vertices of $T_{PT}(T)$ and there are k number of path of pathos of T. Hence the number of vertices in pathos total vertex semientire block graph $T_{PT}(T)$ is (2p + k).

Further by Theorem 1, $E[P_{Tv}(T)]=4q+1+\sum_{1}^{k}v(p_i)+\frac{1}{2}\sum b_i(b_i-1)$. By Remark 1, $T_{PT}(T)=P_{TV}(T)\bigcup P(T)$. Also the number of edges in a path graph P(T) is $\frac{1}{2}\sum \Delta p_i(\Delta p_i-1)$.

Hence the number of edges in total pathos total vertex semientire graph $T_{PT}(T)$ is

$$4q+1+\sum_{1}^{k}v(p_i)+\frac{1}{2}\sum b_i(b_i-1)+\frac{1}{2}\sum \Delta p_i(\Delta p_i-1).$$

Theorem 8.

For any tree T, $T_{PT}(T)$ is always nonseparable.

Proof.

By Remark 1, $T_{PT}(T)$ is the union of $P_{Tv}(T)$ and P(T). By Theorem 3, $P_{Tv}(T)$ is nonseparable. Also in $T_{PT}(T)$, the pathosvertices are adjacent and it form a block. Hence $T_{PT}(T)$ is always nonseparable.

Theorem 9.

For any tree T, total pathos total vertex semientire block graph $T_{PT}(T)$ is not a bipartite graph.

Proof.

Let T be a connected tree then T has at least two edges say,e_1=uv and e_2=vw. Since each edge e_i =b_i is a block and is adjacent to the vertices u and v, also b_i is adjacent to b_j for $i \neq j$ and $j=i+1$. Clearly uvb_i forms a cycle C_3 in $T_{PT}(T)$. By Theorem 4 $T_{PT}(T)$ is not a bipartite graph.

Theorem 10.

For any tree T, $T_{PT}(T)$ is not a complete graph.

Proof.

Let T be any nontrivial connected tree. By the definition of $T_{PT}(T)$, the vertices of T are adjacent to the blockvertices, regionvertices and pathvertices. But for any tree T, the blockvertices are not adjacent to the pathosvertices and the regionvertices in $T_{PT}(T)$. Hence $T_{PT}(T)$ is not a complete graph.

We characterize the graph whose total pathos total vertex semientire block graph and pathos total vertex semientire block graphs are isomorphic.

Theorem 11.

For any tree T, the total pathos total vertex semientire block graph $T_{PT}(T)$ is isomorphic to pathos total vertex semientire block graph $P_{Tv}(T)$ if and only if T is a path P_n for $n \geq 3$.

Proof.

Let T be any nontrivial connected tree and is P_n $n \geq 3$. By the definitions of $P_{Tv}(T)$ and $T_{pv}(T)$, both graphs $P_{Tv}(T)$ and $T_{pv}(T)$ have the same number of vertices. Since T is P_n and it contains only one path. Hence P(T) has no edges. By Remark 1, $T_{PT}(T) = P_{TV}(T) \bigcup P(T)$ and $P(T) = \phi$. This implies by definitions, that $T_{PT}(T)$ and $P_{Tv}(T)$ are isomorphic.

Conversely suppose $T_{PT}(T)$ and $P_{Tv}(T)$ are isomorphic and T be any nontrivial connected tree. We now prove that T is a path P_n. On contrary, assume that T is a tree with at least one vertex $u \in T$ such that $\deg(u) \geq 3.$ But the Remark 1, it is clear that the number of edges in $T_{PT}(T)$ is the sum of the edges in $P_{Tv}(T)$ and the edges in P(T), the path graph. Since T contains at least one vertex u such that $\deg(u) \geq 3$, it follows that P(T) contains at least one edge. Thus the number of edges in $T_{PT}(T)$ is at least one more edge than the number of edge in $P_{Tv}(T)$. Hence total pathos total vertex semientire block graph $T_{PT}(T)$ is not isomorphic to pathos total vertex semientire block graph $P_{Tv}(T)$, a contradiction. Thus T must be a path P_n.

Theorem 12.

For any tree T, the total pathos total vertex semientire block graph $T_{PT}(T)$ is isomorphic to total pathos vertex semientire block graph $T_{Pv}(T)$ if and only if T is a path P_2.

Proof.

Let T be a connected tree without isolated vertices and $T=P_2$. By the definition, both $T_{PT}(T)$ and $T_{Pv}(T)$ have the same number of vertices . For $T=P_2$, the pathos graph P(T) and the block graph B(T) has no edges. Hence the total pathos total vertex semientire block graph $T_{PT}(T)$ is isomorphic to total pathos vertex semientire block graph $T_{Pv}(T)$.

Conversely suppose $T_{PT}(T) \cong T_{Pv}(T)$ and T be any nontrivial tree. We now prove that T is P_2. On contrary, assume T is a path P_n, $n \geq 3.$ Clearly T contains only one path and there is no edge in P(T). in B(T), there are n – 2 edges. Hence the number of edges of $T_{PT}(T)$ is not same as the number of edges of $T_{Pv}(T)$. Thus $T_{PT}(T) \neq T_{Pv}(T)$, a contradiction. Hence T must be a path P_2.

Theorem 13.

For any tree T, total pathos total vertex semientire block graph $P_{Tv}(T)$ is always nonplanar.

Proof.

By the Remark 1, $T_{PT}(T) = P_{TV}(T) \bigcup P(T)$ and by Theorem 6, $P_{Tv}(T)$ is nonplanar hence $T_{PT}(T)$ is nonplanar.

Theorem 14.

For any tree T, $T_{PT}(T)$ always noneulerian.

Proof.

Let T be any tree. We have the following cases.

Case 1.

Suppose T be a star $K_{1,n}$. If n is even, then T contains odd vertices. In $T_{PT}(T)$, the regionvertex is adjacent to odd number of vertices. Hence $T_{PT}(T)$ is noneulerian. If n is odd, then there are $\left\lceil \frac{n}{2} \right\rceil$ path of pathos in T. Clearly all $\left\lceil \frac{n}{2} \right\rceil - 1$ path of pathos contains three vertices and the remaining one path of pathos contains two vertices. In $T_{PT}(T)$, all $\left\lceil \frac{n}{2} \right\rceil - 1$ pathosvertices are adjacent to three vertices to form a graph with odd degree and all pathosvertices adjacent each other to form a complete graph $K_{\left\lceil \frac{n}{2} \right\rceil}$ degree $\left\lceil \frac{n}{2} \right\rceil - 1$. Clearly the path of pathos p_i having odd degree. Hence $T_{PT}(T)$ is noneulerian.

Case 2.

Suppose T be any non-star tree. For sake of simplicity, we consider a tree with vertex of degrees either 1, 2, 5,6,9,10,….. By Remark 3. The corresponding vertices in $T_{PT}(T)$ have even degree. But in this graph at least one path of pathos must contains three vertices and is adjacent to two path of pathos. By definition of $T_{PT}(T)$, the corresponding pathosvertices have odd degree. Hence $T_{PT}(T)$ is always noneulerian.

In the next Theorem we give graph whose total pathos total vertex semientire block graph is the Hamiltonian.

Theorem 15.

For any tree T, $T_{PT}(T)$ is always Hamiltonian.

Proof.

Consider a tree T. We have the following cases.

Case 1.

Suppose T is a star $K_{1,n.}$ Then T has at least one vertex with degree at least three. Assume that T has exactly one vertex u such that $\deg(u) \geq 3$. Now we consider the following sub cases of case 1.

Sub Case 1.1.

Assume that T = $K_{1,n}$, n is odd. Then the number of pathos of pathos are $\frac{n+1}{2} = k$ with pathosvertices p_1, p_2,…, $p_{\frac{n+1}{2}}$. By definition of $T_{PT}(T)$, each of p_{k-1} pathosvertices are adjacent to exactly three vertices and the remaining one path of pathosvertex is adjacent to exactly two vertices. each p_k for all k, path of pathos are adjacent each other. The vertex set of $T_{PT}(T)$ is

$V[T_{PT}(T) = \{u_1, u_2, ..u_n\} \cup \{b_1, b_2, ..b_m\} \cup \{p_1, p_2, ...p_k\} \cup r_1$. Then there exists a cycle r_1 $u_1p_1u_2b_1b_2....b_mu_n...p_ku_{n-1}r_1$, which includes all the vertices of $T_{PT}(T)$. Hence $T_{PT}(T)$ is Hamiltonian as shown in figure 2.4

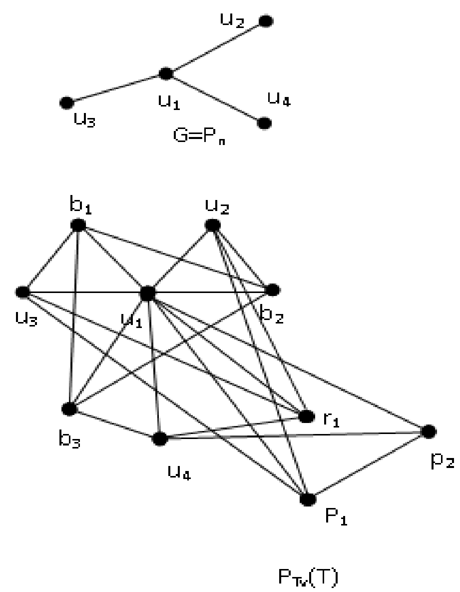


Figure 2.4

Sub Case 1.2.

Assume that T = $K_{1,n}$, n is even. Then the number of path of pathos are $\frac{n}{2} = k$ with pathosvertices $p_1, p_2, \ldots, p_k$. By definition of $T_{PT}(T)$, each p_k for all k, path of pathos are adjacent each other. The vertex set of $T_{PT}(T)$ is $V[T_{PT}(T) = \{u_1, u_2, ..u_n\} \cup \{b_1, b_2, ..b_m\} \cup \{p_1, p_2, ...p_k\} \cup r_1$. Then there exists a cycle r_1 $u_1p_1u_2b_1b_2....b_mu_n...p_ku_{n-1}r_1$, which includes all the vertices of $T_{PT}(T)$. Hence $T_{PT}(T)$ is Hamiltonian.

Case 2.

Suppose T be any tree and it has at least two vertices u and v such that $\deg(u), \deg(v) \geq 3.$ By case 1, $T_{PT}(T)$ is Hamiltonian.

CONCLUSIONS

In this paper we obtained the new graph valued function called the total pathos total vertex semientire block graph of a tree. We studied the characterization of graphs whose of total pathos total vertex semientire block graph of a tree is Hamiltonian, nonplanar and noneulerian.

REFERENCES

1. Haray F., Annals of New York, Academy of Sciences, (1977),175, 198.
2. Harrary, F.,Graph Theory, Addison- Wesley Reading Mass, (1969)., p.72, 107.
3. Kulli. V R., On minimally nonouterplanar graphs, proceedings of the Indian National Science Academy, (1975), 41A.
4. Kulli. V. R. and Akka. D. G., Journal mathematical Science, Vol. 14, No. 6, (1980), P. 585- 588.
5. Rajanna N E and Venkanagouda. M. Goudar "Pathos vertex semientire block graph", International Journal of Mathematics Trends and Technology (2014),Vol 7 (2), 103-105.
6. Rajanna N E and Venkanagouda. M. Goudar " Total vertex and total pathos vertex semientire block graph ", Global Journal of Pure and Applied Mathematics,(2016), Vol 12 (1), 1117-1130
7. Sedlacek J., Some properties of interchange graphs. The Graphs and the applications. Academic press, New York (1962).
8. Venkanagouda. M. Goudar, "On Pathos vertex semientire graph of a tree", international journal of applied mathematical research (2012),Vol 1 (4), 666- 670.
9. Venkanagouda. M. Goudar and Rajanna N E " Vertex semientire block graph ", Global Journal of Science Frontier Research (2014), Vol 14 (1) 1-4.
10. Venkanagouda. M. Goudar and Rajanna N E "Pathos total vertex semientire block graph" (submitted).